**Queen’s College 155 Anniversary Quiz**

1. Consider the number **39**.

 The smallest prime and the biggest prime factor of 39 are 3 and 13.

 The prime numbers between 3 and 13 are 3, 5, 7, 11, 13.

 Also, **39** = 3 + 5 + 7 + 11 + 13.

 Find the next number that has each property.

 (Hint: the number is bigger than 100 and it must not be a prime number.)

**1 Ans.**  **155** = $5+7+11+13+17+19+23+29+31$

**2.** The number of vertices of a right prism is greater than the number of vertices of a right pyramid by 1. If the pyramid has **155** faces, find the sum of the number of edges of the two solids.

**2 Ans.** Number of faces of the pyramid = 155

 Number of **slant** **faces** of the pyramid = 154

 Number of edges of all slant faces = 154

 The base of the pyramid is a polygon of 154 sides and therefore has 154 edges.

 Totol number of edges of the pyramid = 154 + 154 = **308**

 Total number of vertices of the pyramid = 155

 Therefore the total number of vertices of the prism = 156

 The base and the top of the prism is a polygon of $\frac{156}{2}=78$ sides.

 Number of edges of the of the prism = $78×3=234$

 Total number of edges of the two solids = $308 +234 = \overline{542}$

**3.** $15!=15×14×13×…×1=1307674368000$

 There are 3 trailing zeros. (Continuous number of zeros in the right side of the number.)

 How many trailing zeros are there in $155!$

**3 Ans.** A trailing zero is formed when a multiple of 5 is multiplied with a multiple of 2. Now all we have to do is count the number of 5’s and 2’s in the multiplication.

 Since a zero is created by $10=2×5$ and there are more factors of 2 than 5 in $155!$ , all we have to do is to count the factor 5 in the product.

 Let’s count the 5’s first. 5, 10, 15, 20, 25 and so on, making a total $\left[\frac{155}{5}\right]=31$ , where $\left[x\right]$ denotes the largest integer smaller or equal to x.

 However there are numbers 25, 50, 75, … which makes up two fives in each of them ($25=5×5)$, these number count up to $\left[\frac{155}{25}\right]=6$

 Similarly, we need to count up numbers which have 3 fives as factors.

 Number of trailing zeros = $\left[\frac{155}{5}\right]+\left[\frac{155}{25}\right]+\left[\frac{155}{125}\right]=31+6+1=\overline{38}$

**4.** Simplify $1\left(1!\right)+2\left(2!\right)+3\left(3!\right)+…+155\left(155!\right)$

**4 Ans.** $1\left(1!\right)+2\left(2!\right)+3\left(3!\right)+…+155\left(155!\right)$

 $=\left[2\left(1!\right)+3\left(2!\right)+4\left(3!\right)+…+156\left(155!\right)\right]$

 $-\left[1!+ 2!+ 3!+… + 155!\right]$

 $=\left[2!+3!+4!+…+156!\right]-\left[1!+ 2!+ 3!+… +155!\right]$

 $=\overline{156!-1}$

**5.** Find the **centre number** of the 155th row the Kordemsky’s triangular array. Find also the sum of all numbers in this row.



**5 Ans.** The last number of the nth row = $n^{2}$

The centre number of the nth row = $n^{2}-n+1$

 The centre number of the 155th row = $155^{2}-155+1=\overline{23871}$

There are (2n – 1) numbers in the rth row.

The first number in the nth row = $n^{2}-\left(2n – 1\right)+1=n^{2}-2n+2$

 The first number in the 155th row = $155^{2}-2\left(155\right)+2=23717$

 Sum of all numbers in the 155th row = $\frac{2n-1}{2}\left(a+I\right)=\frac{2\left(155\right)-1}{2}\left(23717+155^{2}\right)=\overline{7376139}$

**6.** If $f\left(x^{2}-313x\right)=\left(x-155\right)\left(x-156\right)\left(x-157\right)\left(x-158\right)$, find $f\left(x-155^{2}\right)$.

**6 Ans.** $f\left(x^{2}-313x\right)=\left(x-155\right)\left(x-156\right)\left(x-157\right)\left(x-158\right)$

 $=\left[\left(x-155\right)\left(x-158\right)\right]\left[\left(x-156\right)\left(x-157\right)\right]$

 $=\left[x^{2}-313 x+24490\right]\left[x^{2}-313 x+24492\right]$

 Hence, $f\left(x\right)=\left(x+24490\right)\left(x+24492\right)$

 Replace x by $x-155^{2}$,

 $f\left(x-155^{2}\right)=\left(x-155^{2}+24490\right)\left(x-155^{2}+24492\right)$

 $=\overline{\left(x+465\right)\left(x+467\right)}$

**7.** The sequence $x\_{1},x\_{2},x\_{3},…,x\_{155},x\_{156},…$ satisfies :

 $x\_{1}=\frac{1}{2}, x\_{k+1}=x\_{k}^{2}+x\_{k}$ where k = 1, 2, …, 155,….

 Find the integral part (that is, excluding the decimal part) of the sum

 $\frac{1}{x\_{1}+1}+\frac{1}{x\_{2}+1}+…+\frac{1}{x\_{155}+1}$. (Hint: $\frac{1}{x\_{1}}-\frac{1}{x\_{1}+1}$.)

**7 Ans.** We have $\frac{1}{x\_{1}}-\frac{1}{x\_{1}+1}=\frac{1}{x\_{1}\left(x\_{1}+1\right)}=\frac{1}{x\_{2}}$

 Similarly, $\frac{1}{x\_{2}}-\frac{1}{x\_{2}+1}=\frac{1}{x\_{3}}$

 Work up to $\frac{1}{x\_{154}}-\frac{1}{x\_{155}+1}=\frac{1}{x\_{156}}$

 Add all these identities up, we have $\frac{1}{x\_{1}+1}+\frac{1}{x\_{2}+1}+…+\frac{1}{x\_{155}+1}=\frac{1}{x\_{1}}-\frac{1}{x\_{156}}=2-\frac{1}{x\_{156}}$

 Since $x\_{1}=\frac{1}{2}, x\_{2}=\frac{1}{4}+\frac{1}{2}=\frac{3}{4}, x\_{3}=\left(\frac{3}{4}\right)^{2}+\frac{3}{4}=\frac{21}{16}>1$, we have

 $x\_{1}$<$x\_{2}<x\_{3}<…<x\_{156}$ (More serious reader may use mathematical induction.)

 Then $1=2-1<2-\frac{1}{x\_{156}}<2-0=2$

 The integral part of $\frac{1}{x\_{1}+1}+\frac{1}{x\_{2}+1}+…+\frac{1}{x\_{155}+1}$ is **1**.

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